Math 20550 - Summer 2016

Directional Derivatives and Geometric Uses of the Gradient

June 24, 2016

Problem 1. *Find the directional derivative of* $f(x,y) = \sin(x+2y)$ *at the point* $(0,0)$ *in the direction of* $\mathbf{u} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$.

Problem 2. *Find the directional derivative of* $f(x,y) = x^3 - y^3$ *at the point* $(4,3)$ *in the direction of* $\mathbf{v} = \langle 1, 1\rangle$.

Problem 3. *Find the directional derivative of* $f(x,y,z) = xy + yz + xz$ *at the point* $(1,2,-1)$ *in the direction of* $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

Problem 4. *Find the maximum rate of change of* $f(x,y) = x^2 + 2xy$ *at the point* $(1,0)$ *and give the direction in which it occurs.*

Problem 5. *What is the minimum rate of change of* $f(x,y) = x^2 + 2xy$ *at* $(1,0)$*? In what direction does it occur?*

Problem 6. *Find a unit vector* $\mathbf{w}$ *which is perpendicular to* $\nabla f(1,0)$ *and compute* $D_{\mathbf{w}}f(1,0)$. *What is the geometric meaning of your answer (beyond just saying it's the slope of the tangent line in that direction)?*

Problem 7. *Sketch the level curves* $f(x,y) = c$ *for* $c = 1$ *and* $c = 3$ *where* $f(x,y) = xy$. *WITHOUT COMPUTING THE GRADIENT, sketch the vectors* $\nabla f(1,1)$, $\nabla f(-2,-0.5)$, *and* $\nabla f(1.5,2)$.

Problem 8. *Find the tangent line to the ellipse* $9x^2 + 4y^2 = 40$ *at the point* $(2,-1)$. *Use Calc 3 to do this, do not solve for* x *or* y.

Problem 9. *The tangent plane to a level surface* $f(x,y,z) = k$ *at the point* P *is given by* $x - 2z = 3$. *Find a unit vector which is parallel to the gradient vector,* $\nabla f(P)$.

Problem 10. *Find the tangent plane and normal line to the surface* $y \ln xz^2 = 2$ *at the point* $(e,2,1)$.

Problem 11. *Find the tangent line to the curve of intersection of* $x^2 + y^2 = 1$ *and* $z = 3xy$ *at the point* $(0,-1,0)$.

Problem 12. *At what points do the surfaces* $x^2 + y^2 - z^2 = 1$ *and* $x^2 + 4y^2 + 4z^2 = 1$ *intersect tangentially?*